AIAA JOURNAL
Vol. 37, No. 10, October 1999

Technical Notes

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Improved Formulation for Geometric Properties of Arbitrary Polyhedra

Z. J. Wang*
CFD Research Corporation, Huntsville, Alabama 35805

Introduction

IT is necessary to compute various geometric properties in a finite volume discretization scheme used in computational fluid dynamics. These properties often include face areas and normal vectors, face centers, cell volumes and cell centroids, etc. In a computational grid composed of relatively simple cell types, such as tetrahedra, hexahedra, and prisms, it is relatively easy to compute these properties. For a computational grid with arbitrary polyhedral cells, such as cut cells generated through cell cutting in an adaptive Cartesian grid method,[1] it is not trivial to calculate the geometric properties, especially the cell centroids. In a recent paper by Bruner,[2] a formula was given to compute the cell centroid of an N-faced polyhedron. In Bruner's formula, the cell centroid is computed through a surface integral over the N faces bounding the cell. The surface integral, however, requires the evaluation of a Gaussian quadrature. The computation of the quadrature needs three quadrature points for a triangle and four points for a quadrilateral. The computation of the Gaussian quadrature for an arbitrary polygonal face is not straightforward. In this Note, a new formulation is derived for the cell centroid of an N-faced polyhedron for which each face is an arbitrary planar polygon. The new centroid formulation does not need the evaluation of a Gaussian quadrature. Only surface areas, normals, and face centers are required.

Formulation of Face Properties

For the sake of completeness, we first give the expressions for face geometric properties, such as the face area S, normal vector $\boldsymbol{n}$, and face center $\boldsymbol{r}^c$, where c is the face center, for a three-dimensional arbitrary planar polygon with M edges and M vertices whose position vectors are $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_M$. Because a nonplanar polygon can always be decomposed into planar polygons, this assumption is used without loss of generality. The polygon is first decomposed into $M-2$ triangles, assuming that all of the triangles share the first node $\boldsymbol{r}_1$. Then the face area vector ($\boldsymbol{S} = S\boldsymbol{n}$) is simply

$$\boldsymbol{S}_{1,2,\ldots,M} = \sum_{i=2}^{M-1} \boldsymbol{S}_{1,i,i+1} \tag{1}$$

where $\boldsymbol{S}_{1,2,\ldots,M}$ is the area vector of the polygon. The face center for a triangle $(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)$ is simply

$$\boldsymbol{r}^c_{1,2,3} = \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3}{3} \tag{2}$$

Therefore, the center for the polygon is computed using

$$\boldsymbol{r}^c_{1,2,\ldots,M} = \left(\sum_{i=2}^{M-1} S_{1,i,i+1}\boldsymbol{r}^c_{1,i,i+1}\right) \Bigg/ \left(\sum_{i=2}^{M-1} S_{1,i,i+1}\right) \tag{3}$$

Formulation for Cell Properties

The expression for the volume computation is the same as shown in Ref. 2, that is, through the use of the divergence theorem from vector calculus:

$$\int_V \nabla \cdot \boldsymbol{v}\,\mathrm{d}V = \int_S \boldsymbol{v}\cdot\boldsymbol{n}\,\mathrm{d}S \tag{4}$$

where $\boldsymbol{v}$ is an arbitrary differential vector. The volume V can be computed by taking $\boldsymbol{v}$ to be the position vector $\boldsymbol{r}$, that is,

$$V = \frac{1}{3}\sum_{i=1}^{N} \boldsymbol{r}^c_i \cdot \boldsymbol{n}_i S_i \tag{5}$$

The expression for the centroid is now derived. The centroid of any control volume is given by

$$\boldsymbol{r}_c = \frac{1}{V}\int_V \boldsymbol{r}\,\mathrm{d}V \tag{6}$$

Let us first compute x_c, that is,

$$x_c = \frac{1}{V}\int_V x\,\mathrm{d}V \tag{7}$$

It is obvious that

$$x = \tfrac{1}{3}x\nabla\cdot\boldsymbol{r} = \tfrac{1}{3}[\nabla\cdot(x\boldsymbol{r}) - \boldsymbol{r}\cdot\nabla x] \tag{8}$$

Therefore,

$$x_c V = \frac{1}{3}\int_V \nabla\cdot(x\boldsymbol{r})\,\mathrm{d}V - \frac{1}{3}\int_V \boldsymbol{r}\cdot\nabla x\,\mathrm{d}V \tag{9}$$

However,

$$\boldsymbol{r}\cdot\nabla x = (x, y, z)\cdot(1, 0, 0) = x \tag{10}$$

Then Eq. (9) becomes

$$x_c V = \frac{1}{3}\int_V \nabla\cdot(x\boldsymbol{r})\,\mathrm{d}V - \frac{1}{3}\int_V x\,\mathrm{d}V = \frac{1}{3}\int_V \nabla\cdot(x\boldsymbol{r})\,\mathrm{d}V - \frac{1}{3}x_c V \tag{11}$$

From Eq. (11) it is then obvious that

$$4x_c V = \int_V \nabla\cdot(x\boldsymbol{r})\,\mathrm{d}V \tag{12}$$

Applying the divergence theorem, we obtain

$$\int_V \nabla\cdot(x\boldsymbol{r})\,\mathrm{d}V = \int_S x\boldsymbol{r}\cdot\boldsymbol{n}\,\mathrm{d}S \tag{13}$$

For an arbitrary planar polygonal face, the term $\boldsymbol{r}\cdot\boldsymbol{n}$ is a constant. Therefore, we have

$$\int_S x\boldsymbol{r}\cdot\boldsymbol{n}\,\mathrm{d}S = \sum_{i=1}^{N} \boldsymbol{r}^c_i\cdot\boldsymbol{n}_i \int_{S_i} x\,\mathrm{d}S = \sum_{i=1}^{N} \boldsymbol{r}^c_i\cdot\boldsymbol{n}_i x^c_i S_i \tag{14}$$

Received 2 February 1998; accepted for publication 4 June 1999.

*Group Leader, Research, 215 Wynn Drive; zjw@cfdrc.com.

Substituting Eqs. (14) and (5) into Eq. (12), we obtain the following formula for x_c:

$$x_c = \frac{3}{4}\left(\sum_{i=1}^{N} \boldsymbol{r}_i^c \cdot \boldsymbol{n}_i x_i^c S_i\right) \Bigg/ \left(\sum_{i=1}^{N} \boldsymbol{r}_i^c \cdot \boldsymbol{n}_i S_i\right) \tag{15}$$

The centroid position vector can be written in the following vector form:

$$\boldsymbol{r}_c = \frac{3}{4}\left[\sum_{i=1}^{N} \left(\boldsymbol{r}_i^c \cdot \boldsymbol{n}_i\right)\boldsymbol{r}_i^c S_i\right] \Bigg/ \left[\sum_{i=1}^{N} \boldsymbol{r}_i^c \cdot \boldsymbol{n}_i S_i\right] \tag{16}$$

Conclusions

A new formulation for the centroid of an arbitrary polyhedron bounded by arbitrary planar polygonal faces is derived. Compared with an earlier formulation, the new formula does not need numerical integrations using Gaussian quadratures over the faces. Only face properties such as areas, unit normals, and face centers are employed to compute the cell centroid. The implementation of the new formula is, therefore, straightforward. The extension of the formula to polyhedra bounded by nonplanar polygons can be accomplished by decomposing the nonplanar polygons into planar ones. Therefore, the formulation can be used to compute geometric properties for arbitrary polyhedra.

References

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J. Kallinderis
Associate Editor

Euler/Navier–Stokes Optimization of Supersonic Wing Design Based on Evolutionary Algorithm

Akira Oyama,* Shigeru Obayashi,† and Kazuhiro Nakahashi‡
Tohoku University, Sendai 980-8579, Japan
and
Takashi Nakamura§
National Aerospace Laboratory, Tokyo 182-8522, Japan

Introduction

APPLICATION of numerical optimization to aerodynamic design is a difficult task. In Ref. 1, it was reported that the distribution of the objective function could be extremely multimodal even in a simplified problem. In addition, function evaluations using a computational fluid dynamics (CFD) code, especially an Euler or Navier–Stokes code, are very expensive. Therefore, both an optimization algorithm with high parallel efficiency and a powerful parallel computer are required to accomplish aerodynamic optimization.

Among optimization algorithms, evolutionary algorithms[2] (EAs) appeal to many designers and researchers because of their robustness. High parallel efficiency is also achieved by using a simple master–slave concept for function evaluations if such evaluations consume most of the CPU time. Aerodynamic optimization using CFD will be used as a typical case.

The purpose of this study is to examine the feasibility of supersonic wing design optimization using EAs coupled with Euler/Navier–Stokes computation. To overcome the expected difficulty in CPU time, computation is performed on a parallel vector machine called the numerical wind tunnel[3] (NWT). Grid generation and flow calculation of each design candidate are distributed to 64 processing elements (PEs), whereas EA operators are assigned to the master computer because their CPU time is negligible.

Formulation of Optimization Problem

In this study, an aerodynamic shape of a supersonic wing is optimized at the supersonic cruise design point. The cruising Mach number is set to 2.3. The purpose of the present study is to maximize the lift-to-drag ratio L/D, maintaining substantial lift coefficient C_L and wing thickness. The optimization problem is defined as follows: The objective function to be maximized is L/D with the constraints $C_L = 0.1$ and thickness to chord $t/c \geqq 0.35$.

The lift constraint is satisfied by changing the geometric angle of attack at the wing root so that C_L becomes 0.1 based on the lift coefficient varying linearly. This approach requires two extra flow evaluations.

The aerodynamic performance is evaluated by using an Euler/Navier–Stokes code. This code employs total variation diminishing-type upwind differencing,[4] the lower–upper symmetric Gauss–Seidel scheme, and the multigrid method.

Airfoil sections of design candidates are generated by the extended Joukowski transformation.[5] It transforms a circle to various kinds of airfoils by two consecutive conformal mappings using five parameters: x_c, y_c, x_t, y_t, and Δ. Airfoil sections defined by these extended Joukowski parameters and the twist angle will be given at eight span sections; spanwise locations are also treated as design variables except for the wing root and tip locations. Wing geometry is then interpolated in the spanwise direction by using the second-order spline interpolation. The planform is assumed to be a double-delta wing similar to the National Aerospace Laboratory scaled supersonic experimental airplane.

Optimization Using EA

In the present EA, design variables are coded in finite length strings of real numbers corresponding to the five Joukowski transformation parameters, the twist angle, and their spanwise locations. Fitness of an individual is determined by its rank among the population based on its L/D. Selection is performed by the stochastic universal sampling[6] coupled with the elite strategy. Ranking selection is adopted because it maintains sufficient selection pressure throughout the optimization. Then the offspring (the new design candidates) are produced, applying a one-point crossover and an evolutionary direction operator[7] half-and-half to the mating pool (selected design candidates). During the reproduction process, mutation takes place at a probability of 20% and then adds a random disturbance to the corresponding gene. The population size is kept at 64.

To reduce the wall clock time necessary for this optimization, evaluations using the Euler/Navier–Stokes code are distributed to 64 PEs of the NWT. Because the CPU time used for EA operators is negligible, turnaround time becomes almost $\frac{1}{64}$. Actually, whereas each CFD evaluation took about 1 h of CPU time (for three Euler evaluations) on the slave PE, the EA operators took less than 1 s on the master PE.

Results

Because the wing planform is fixed and the viscous drag primary depends on the planform area, inviscid calculations are used for the present evaluations. The total drag evaluated here consists of the

Received 22 February 1999; accepted for publication 9 June 1999.

*Graduate Student, Nakahashi Laboratory, Department of Aeronautics and Space Engineering, Aoba-yama 01.

†Associate Professor, Department of Aeronautics and Space Engineering, Aoba-yama 01. Senior Member AIAA.

‡Professor, Department of Aeronautics and Space Engineering, Aoba-yama 01. Senior Member AIAA.

§Senior Researcher, Computational Science Division, Chofu.